

# **Hedge Fund Replication Seminar**

*Financial Engineering Seminar, October 15th, 2007*

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## **Hedge Fund Replication – An Assessment of Existing Techniques and Directions for Further Research**

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Joint work with Noël Amenc & Volker Ziemann

# Overview

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- The Myths & Limits of HF Replication
- Payoff-Distribution Approach to HF Replication
- Factor-Based Approach to HF Replication: the Linear Case
- Factor-Based Approach to HF Replication: the Non-Linear Case
- Factor-Based Approach to HF Replication: Conditional Models
- Factor-Based Approach to HF Replication: Economic Models
- Conclusion & Directions for Further Research
- Appendix

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# The Myths and Limits of HF Replication

## *Beta versus Alpha Drivers*

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- HF returns basic decomposition:

$$\text{HF Returns} = \text{Traditional Factor Exposures} + \text{Alternative Factor Exposures} + \text{Alpha}$$

- Following recent initiatives by major investment banks such as Merrill Lynch and Goldman Sachs, there has been a renewed interest in the financial industry concerning the subject of passive hedge fund replication, an ancient subject of interest in academic circles.
- The (probably not exhaustive) list of hedge fund replication product providers currently includes: *AlphaSwiss, Barclays, Bear Stearns, Deutsche Bank, Goldman Sachs, IceCapital Fund Management Company, IndexIQ, JP Morgan, Morgan Stanley, Merrill Lynch, Partners Group, Rydex Investments, SGAM Alternative Investments, State Street Global Advisors, Stonebrook Capital Management LLC.*

# The Myths and Limits of HF Replication

## *Replicating Beta Components of HF Performance*

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- In a nutshell, these initiatives are meant to replicate the beta component of hedge fund returns through investment in a set of rules-based passive strategies.
- Replicating the systematic components of hedge fund returns is a priori an attractive value proposition because:
  - Beta benefits of hedge funds may be more reliable and in the end more attractive to investors than their alpha benefits.
  - Passive replication involves allows investors to avoid the main drawbacks of investment in active hedge funds of funds of funds : lack of liquidity, double-digit fee structure, lockup period, expensive due diligences, etc.
- In what follows, we provide a critical discussion of state-of-the-art practices in hedge fund replication.

# The Myths and Limits of HF Replication

## *Performance versus Payoff Replication*

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- There are two possible definitions for what “passive replication of hedge fund returns” actually means.
- The first definition is replicating *hedge fund performance* on a monthly basis.
- The second definition is replicating the *distribution of hedge fund returns* at some time-horizon.
- If one can replicate the time-series performance (definition 1), one also replicates the distributional properties (definition 2), but the converse is not true.

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# Payoff Distribution Approach

## *Basic Principles*

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- The approach, known as the payoff-distribution approach, and arguably not the most straightforward way to tackle the problem, merely aims at replicating hedge fund return distributions.
- It has first been first developed by Amin and Kat (2003) in a performance measurement context.
- It is based on a (weaker) re-interpretation of the replication concept.

$$\Pr(R_t^{Clone} = R_t^{HF}) = 1 \quad (1) \quad \Pr(R_t^{Clone} < x) = \Pr(R_t^{HF} < x) \quad (2)$$

- In (1), the two variables are said equal almost surely; In (2), the variables are said equal in distribution.
- (1) implies (2) but (2) does not imply (1).

# Payoff Distribution Approach

## *Basic Principles*

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- The process is simple and based on the following three-steps process:
  - First: consider some underlying asset (called reserve or reference asset) and estimate the payoff function that maps the asset return into a hedge fund return.
  - Second: price this payoff function
  - Third: derive the replicating strategy
- After all, the only new thing consists in creating a payoff function that is synthetic.

# Payoff Distribution Approach

## *Basic Principles*

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- To obtain the payoff function  $g$ , we note that:

$$\Pr(HF_T \leq y) = \Pr(g(Index_T) \leq y) = \Pr(Index_T \leq g^{-1}(y))$$
$$\Rightarrow F_{hf}(y) = F_{Index}(g^{-1}(y)) \Rightarrow y = F_{hf}^{-1}[F_{Index}(g^{-1}(y))]$$

- Using

$$y = g(x)$$

$$x = g^{-1}(y)$$

we finally obtain  $g(x) = F_{hf}^{-1}(F_{index}(x))$

- For example, assume there is a 10% probability that an index return over a horizon  $T=1$  year takes on a value higher than 20%; hence, we have that:

$$\Pr(Index_T \leq 20\%) = F_{index}(20\%) = 0.9$$

- We then define  $g(20\%)$  as the value that is such that the hedge fund return to be replicated has only a 10% chance of being higher, or:

$$g(20\%) = y \text{ such that } \Pr(HF_T \leq y) = F_{hf}(y) = 0.9$$

# Payoff Distribution Approach

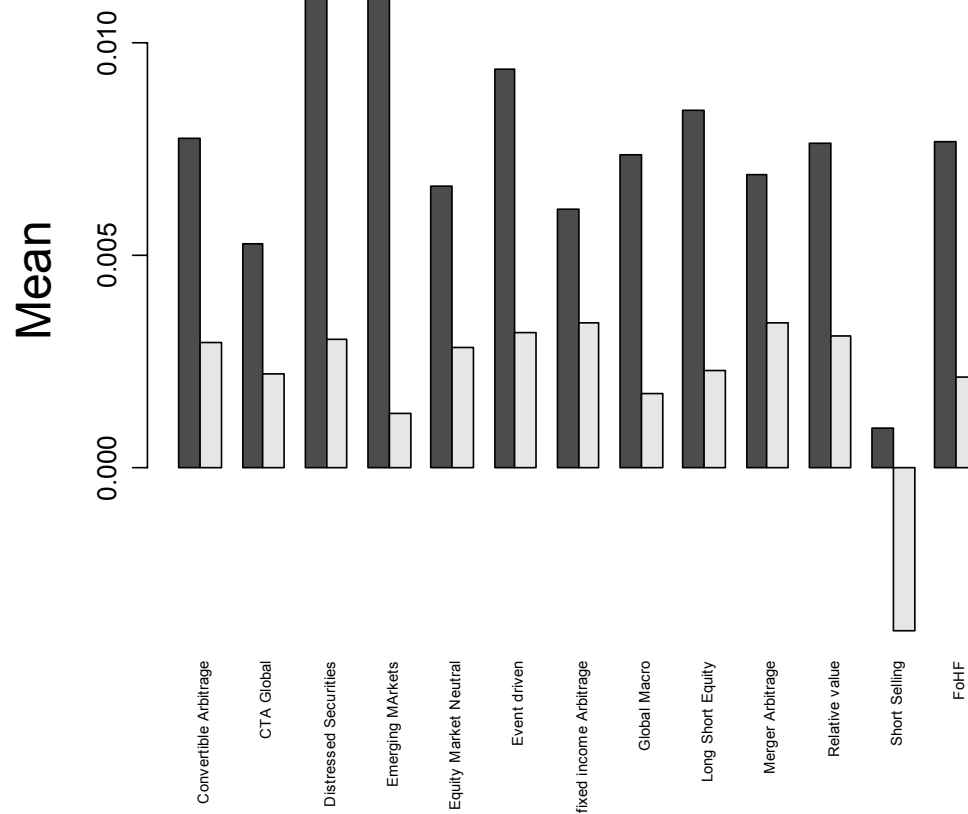
## *Empirical Results*

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- We have decided to get a better feel for the performance of their methodology.
- Application to 13 Edhec hedge fund indices between 01/1997 to 12/2006.
- We use nearby futures contract on the S&P500 index as the reserve asset and the 3 months Eurodollar as a proxy for the risk free rate.
- For each monthly date  $t$  starting as of January 1999, we construct the payoff distribution on a sample period ranging from January 1997 the current date (excluded).
- We use Monte-Carlo analysis to price the payoffs and derive the replicating strategies (delta).

# Payoff Distribution Approach

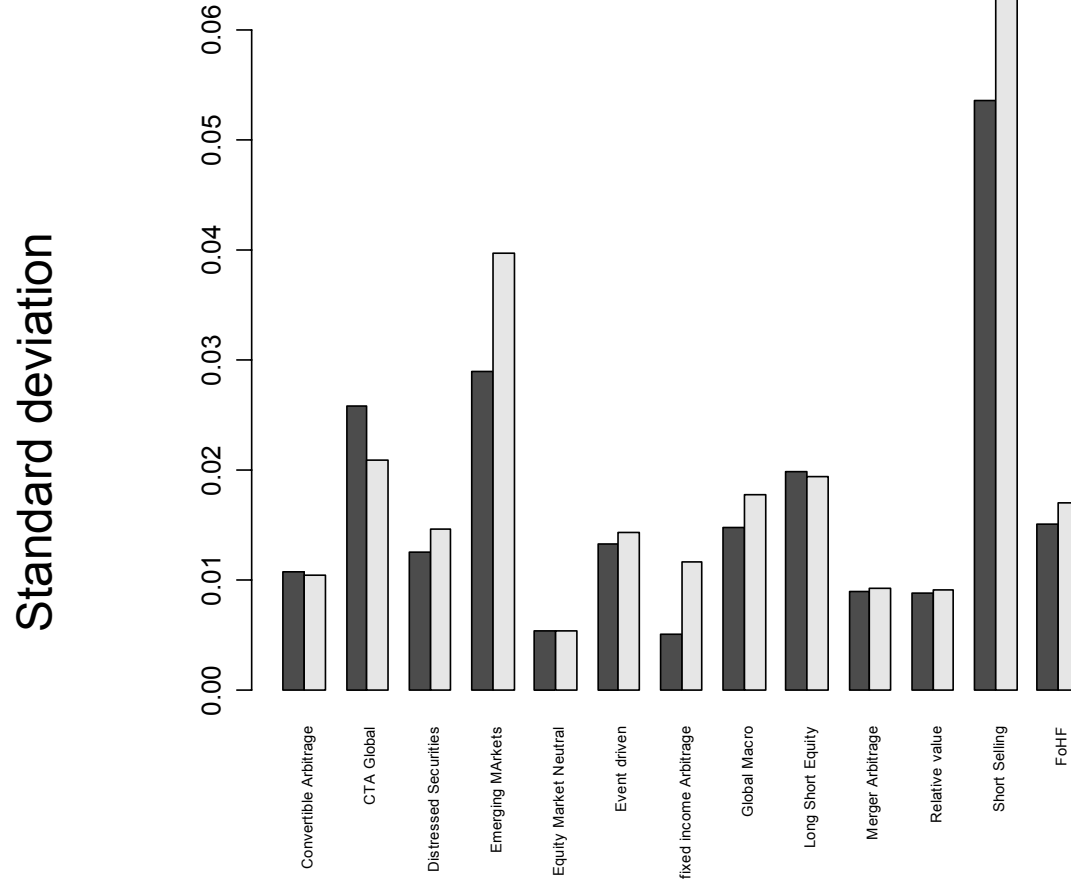
## *Empirical Results*



- Average performance of the clones is much lower than average performance of indices.
- This is contrast to Amin and Kat, who focus on a time-period that does not include the bear equity market.
- Performance (arguably an important question for the investor!) is not controlled by the method (only higher moments are).

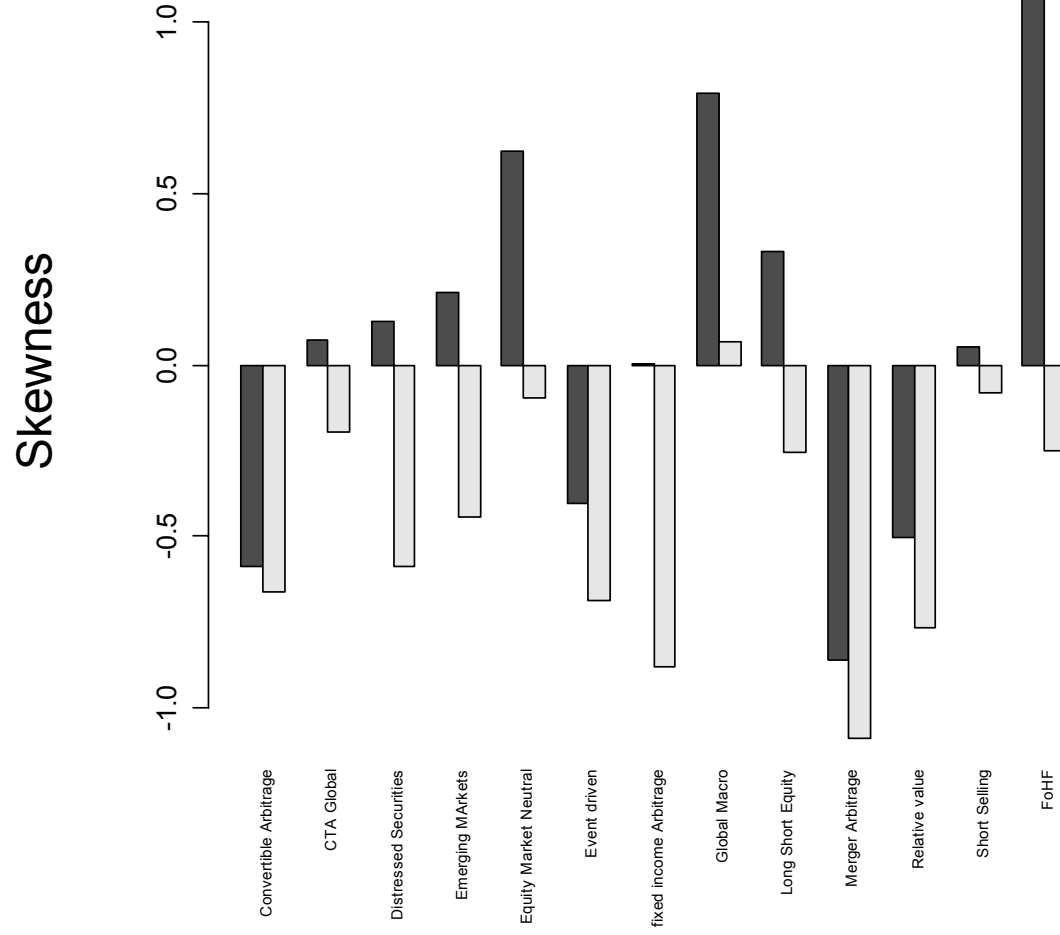
# Payoff Distribution Approach

## *Empirical Results*



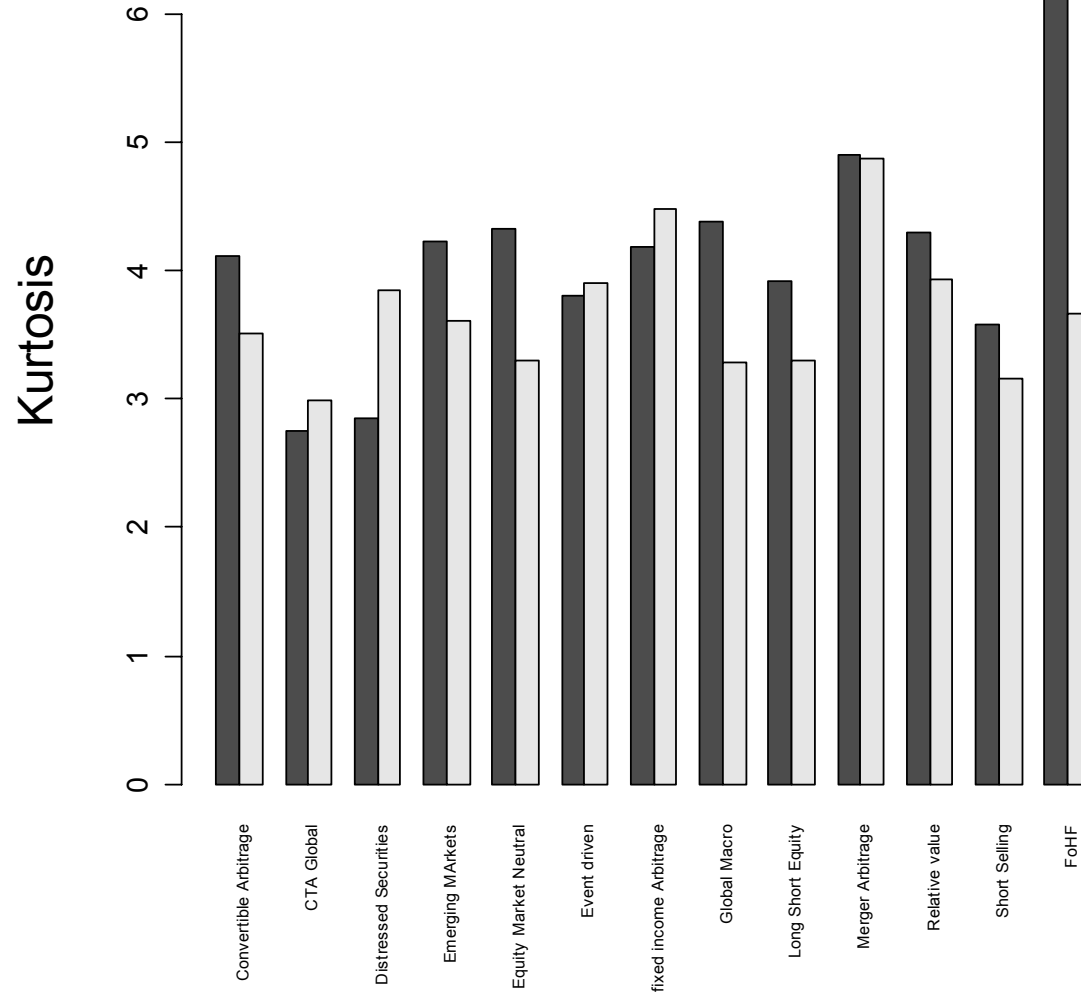
# Payoff Distribution Approach

## *Empirical Results*



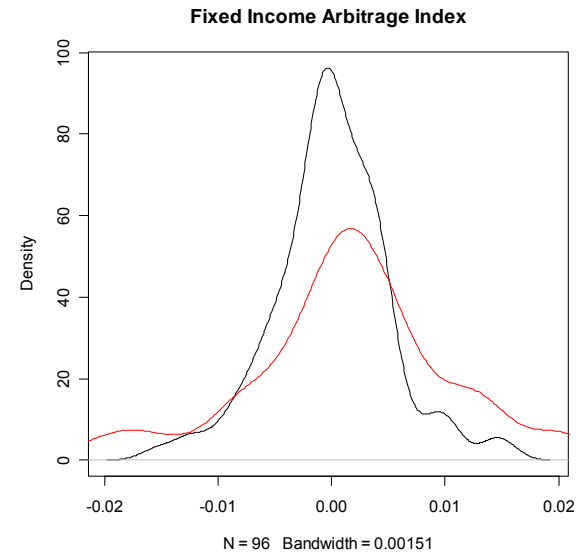
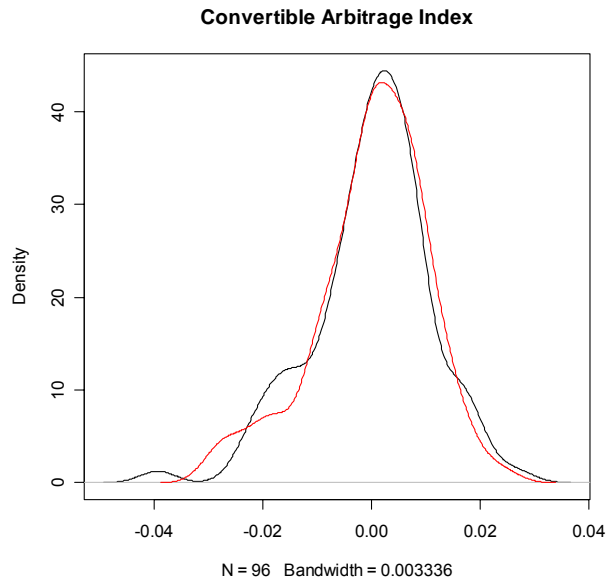
# Payoff Distribution Approach

## *Empirical Results*



# Payoff Distribution Approach

## *Empirical Results*



Best and worst distribution fits.

Watch out: distributions have been centered – mean performance can vary a lot!

# Payoff Distribution Approach

## *Empirical Results*

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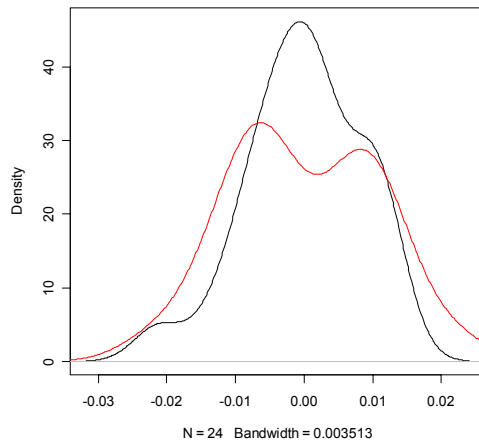
- Those results were obtained when a long out of sample period is used (96 monthly observations corresponds to 8 years!)
- The same methodology for a smaller out-of-sample period (01/1999 to 12/2000) leads to a rejection of the Merger Arbitrage and Relative Value indices in addition to the Fixed-Income Arbitrage index.
- We need a patient investor!

# Payoff Distribution Approach

## *Empirical Results*

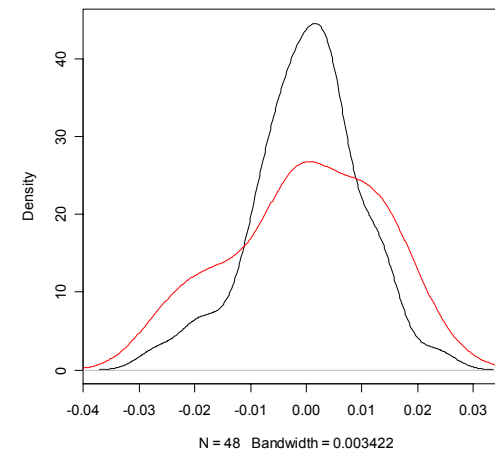
01/1999 to 12/2000

Convertible Arbitrage Index



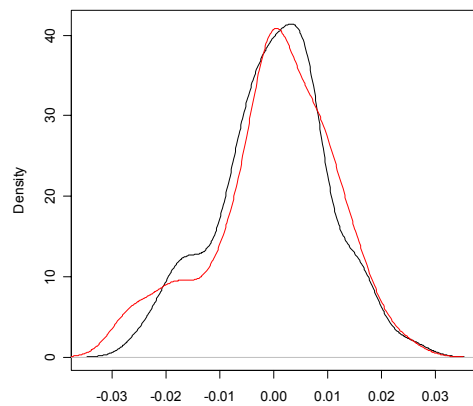
01/1999 to 12/2002

Convertible Arbitrage Index



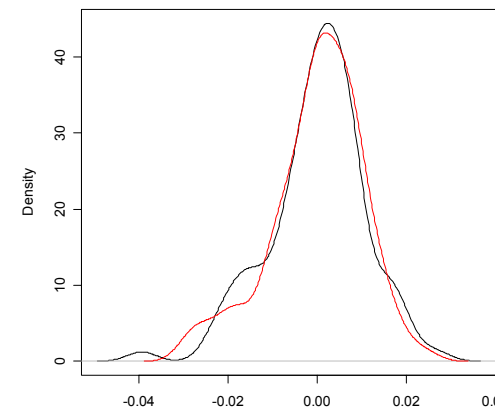
01/1999 to 12/2004

Convertible Arbitrage Index



01/1999 to 12/2006

Convertible Arbitrage Index



# Payoff Distribution Approach

## *Empirical Results – Time-Series Properties*

	Correlation between the clone and the index	R-squared of a regression of the clone return on the index return
Convertible Bond Arbitrage	0.1402	1.97%
CTA global	-0.1782	3.18%
Distressed Securities	0.3701	13.70%
Emerging markets	0.6233	38.85%
Equity market neutral	0.3106	9.65%
Event driven	0.5861	34.35%
Fixed income arbitrage	0.0886	0.78%
Global macro	0.3253	10.58%
Long/short Equity	0.6740	45.43%
Merger arbitrage	0.5369	28.83%
Relative value	0.6555	42.97%
Short selling	- 0.7495	56.18%
Funds of funds	0.4725	22.33%



# Payoff Distribution Approach

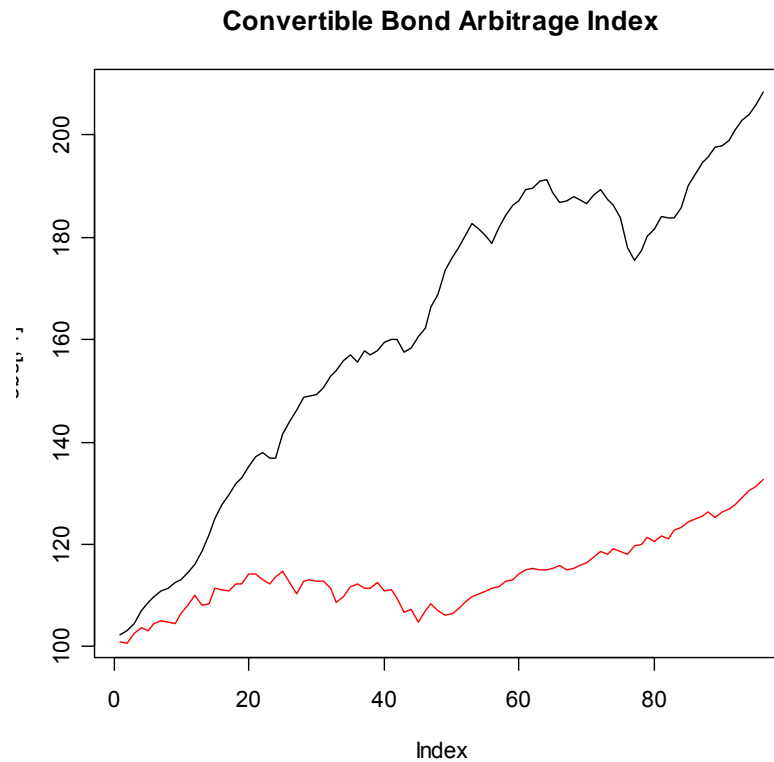
## *Empirical Results Time-Series Properties*

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- Monthly differences between replicated values and actual prices are too large to make this approach suitable for an investor aiming at replicating hedge fund returns.
- 6 indices exhibit a monthly difference higher than [-50 bps; +50 bps]!
- We also test the evolution of \$100 invested in both the Convertible Arbitrage index and in its clone.

# Payoff Distribution Approach

## *Empirical Results*



Evolution of the difference in time series between a synthetic and real investment in the Convertible Bond Arbitrage Index

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# Factor-Based Approach – Linear Case

## *Basic Principles*

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- The most straightforward approach to hedge fund replication is based on factor models.
- Aim is to find long/short positions in a set of suitably-selected risk factors that best approximate hedge fund returns, which reads in the linear case:

$$R_t^{HF_i} = \sum_{k=1}^K \beta_{ik} F_{kt} + \varepsilon_{it}$$

- The next step is just to passively held the mimicking portfolio in an out-of-sample period:

$$\hat{R}_t^{Clone_i} = \sum_{k=1}^K \hat{\beta}_{ik} F_{kt}$$

- The performance of these models is moderate on an in-sample basis and poor on an out-of-sample.

# Factor-Based Approach – Linear Case

## *Main Hurdles*

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- Concern over misspecification in the choice of factors.
  - ⇒ specification risk
- Results are very sensitive to the presence of outliers.
  - ⇒ sample risk
- Need for non-linear models: linear factor model can not satisfactorily capture hedge fund risk exposure.
  - ⇒ model risk
- Need for conditional models: factor exposure of hedge fund managers is time-varying.
  - ⇒ stationarity risk

# Factor-Based Approach – Linear Case

## *Literature Review*

Authors	Strategies	ABS/RBS	(In-sample period ) In-sample results	(Out-of-sample period ) Out-of-sample results
Agarwal, Fung, Loon, Naik (2005)	Convertible Arbitrage	ABS	R <sup>2</sup> 14.67% to 55.95%	n/a
Agarwal, Naik (2004)	Convertible Arbitrage Equity Long/Short Equity Non Hedge Event Driven Relative Value Restructuring Risk Arbitrage Short Selling	ABS	(1990-2000) R <sup>2</sup> 40.50% R <sup>2</sup> 72.50% R <sup>2</sup> 91.63% R <sup>2</sup> 73.40% R <sup>2</sup> 52.20% R <sup>2</sup> 65.60% R <sup>2</sup> 44% R <sup>2</sup> 82%	(2000-2001) approximate replication of index returns (standard t-test & Wilcoxon sign-test)
Fung, Hsieh (2002)	Convertible Bond High-Yield Mortgage-Backed Arbitrage Diversified	RBS/ABS	(1990-2000) R <sup>2</sup> 70% to 75% R <sup>2</sup> 78% to 79% R <sup>2</sup> 59% to 66% R <sup>2</sup> 66% R <sup>2</sup> 64%	n/a
Fung, Hsieh (2004)	Multi-strategy portfolio	ABS	(1994-2002) R <sup>2</sup> 55%	n/a
Jaeger, Wagner (2005)	Convertible Arbitrage Distressed Securities Equity Long/Short Equity Market Neutral Event Driven Fixed Income Global Macro Merger Arbitrage Short Selling	ABS	(1994-2004) R <sup>2</sup> 54% R <sup>2</sup> 68.40% R <sup>2</sup> 88.50% R <sup>2</sup> 35.30% R <sup>2</sup> 79.30% R <sup>2</sup> 40.5% R <sup>2</sup> 49.70% R <sup>2</sup> 52.90% R <sup>2</sup> 81.20%	(2003-2005) <i>RFS in % / HFR1 / HFRX</i> +7.6 / +2.4 / -5.3 +20.1 / +44.8 / +23.3 +27.8 / +32.8 / +16 +6.2 / +10.9 / -3.9 +29.8 / +40 / +24.1 +7.8 / +16.3 / n/a +16.7 / +24.6 / +10.1 +13 / +15.3 / +10.9 -28.2 / -23 / n/a

# Factor-Based Approach – Linear Case

## *Empirical Tests*

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- In what follows, we have chosen to reproduce the results reported in Hasanhodzic and Lo (2006).
- The reason why we have chosen to focus on this particular model is essentially related to the fact that it is a simple parsimonious approach that is not suspect of data mining.
- The authors have introduced a factor model based on the following 6 factors for all strategies:
  - US Dollar Index
  - Lehman Corporate AA Intermediate Bond Index
  - Spread between the Lehman Corporate BAA Bond Index and the Lehman Treasury Index
  - S&P 500
  - Goldman Sachs Commodity Index
  - First difference of the end-of-month value of the CBOE Volatility Index

# Factor-Based Approach – Linear Case

## *Empirical Results*

	Average Returns		Standard deviation		Cornish-Fisher-VaR		Sharpe-Ratio (2% p.a.)		Correlation	Info Ratio
	Index	Clone	Index	Clone	Index	Clone	Index	Clone		
Convertible Arbitrage	0.096	0.022	0.037	0.023	0.012	0.010	2.032	0.099	0.108	-1.658
CTA Global	0.061	0.040	0.090	0.068	0.037	0.030	0.457	0.297	0.360	-0.281
Distressed Securities	0.141	0.043	0.043	0.035	0.009	0.012	2.794	0.669	0.267	-1.840
Emerging Markets	0.168	0.019	0.101	0.094	0.032	0.043	1.470	-0.007	0.581	-1.489
Equity Market Neutral	0.082	0.036	0.019	0.010	0.001	0.002	3.358	1.584	0.101	-2.167
Event Driven	0.117	0.045	0.046	0.038	0.014	0.015	2.124	0.644	0.526	-1.612
Fixed Income Arbitrage	0.075	0.009	0.018	0.030	0.002	0.016	3.107	-0.365	0.276	-2.056
Global Macro	0.091	0.026	0.051	0.035	0.013	0.016	1.372	0.168	0.343	-1.196
Long/Short Equity	0.103	0.043	0.069	0.054	0.022	0.024	1.206	0.421	0.663	-1.097
Merger Arbitrage	0.085	0.045	0.031	0.021	0.010	0.006	2.094	1.193	0.294	-1.193
Relative Value	0.095	0.027	0.031	0.025	0.008	0.011	2.446	0.256	0.494	-2.225
Short Selling	-0.006	0.017	0.186	0.185	0.086	0.068	-0.140	-0.014	0.720	0.097
Funds of Funds	0.095	0.035	0.052	0.041	0.011	0.017	1.430	0.366	0.472	-1.158

*Performance and risk indicators for EDHEC Hedge Fund indices and their linear clones based on Hasanhodzic and Lo (2007) are given. All statistics are based on out-of-sample returns observed over the period as of January 1999 through December 2006.*

# Factor-Based Approach – Linear Case

## *Factor-Based Replication*

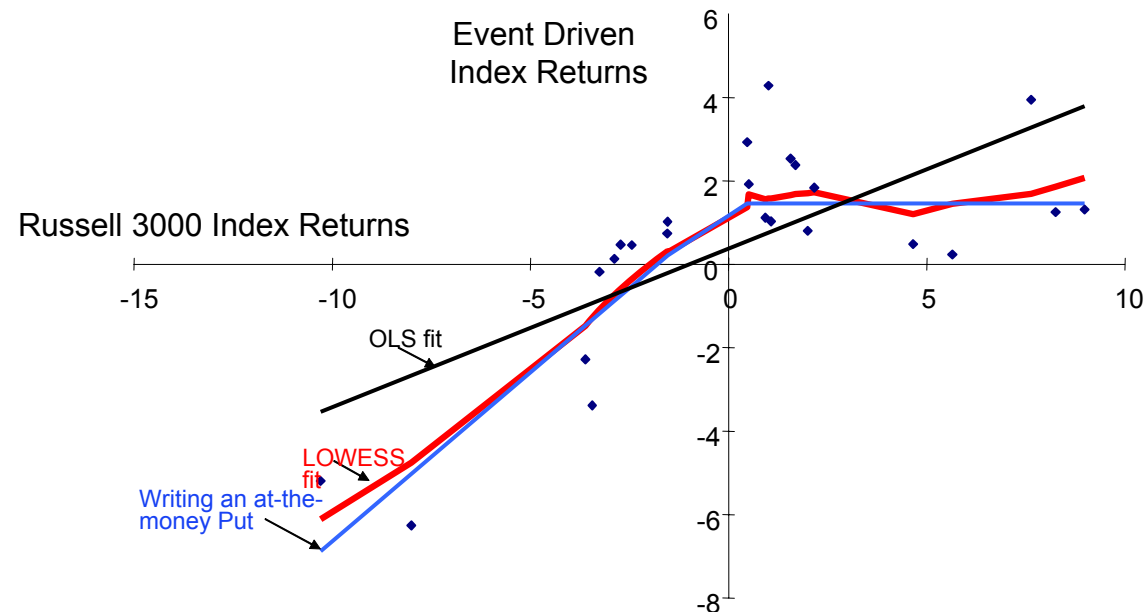
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- Factor-based hedge fund replication models have mostly failed in thorough empirical tests to produce satisfactory results on an out-of-sample basis.
- Because of the non-linear and dynamic exposure of hedge fund returns with respect to underlying risk factors, and in the absence of a true modeling of the time-variations in these factor exposures, simple stepwise linear regression techniques, which simply match the average *past* exposures of hedge fund managers to underlying risk factors, are bound to perform poorly on an out-of-sample basis.
- Factor-approach will not work until we develop satisfactory conditional and/or non-linear factor models, which still is a subject of academic research, including ours.

# Factor-Based Approach – Linear Case

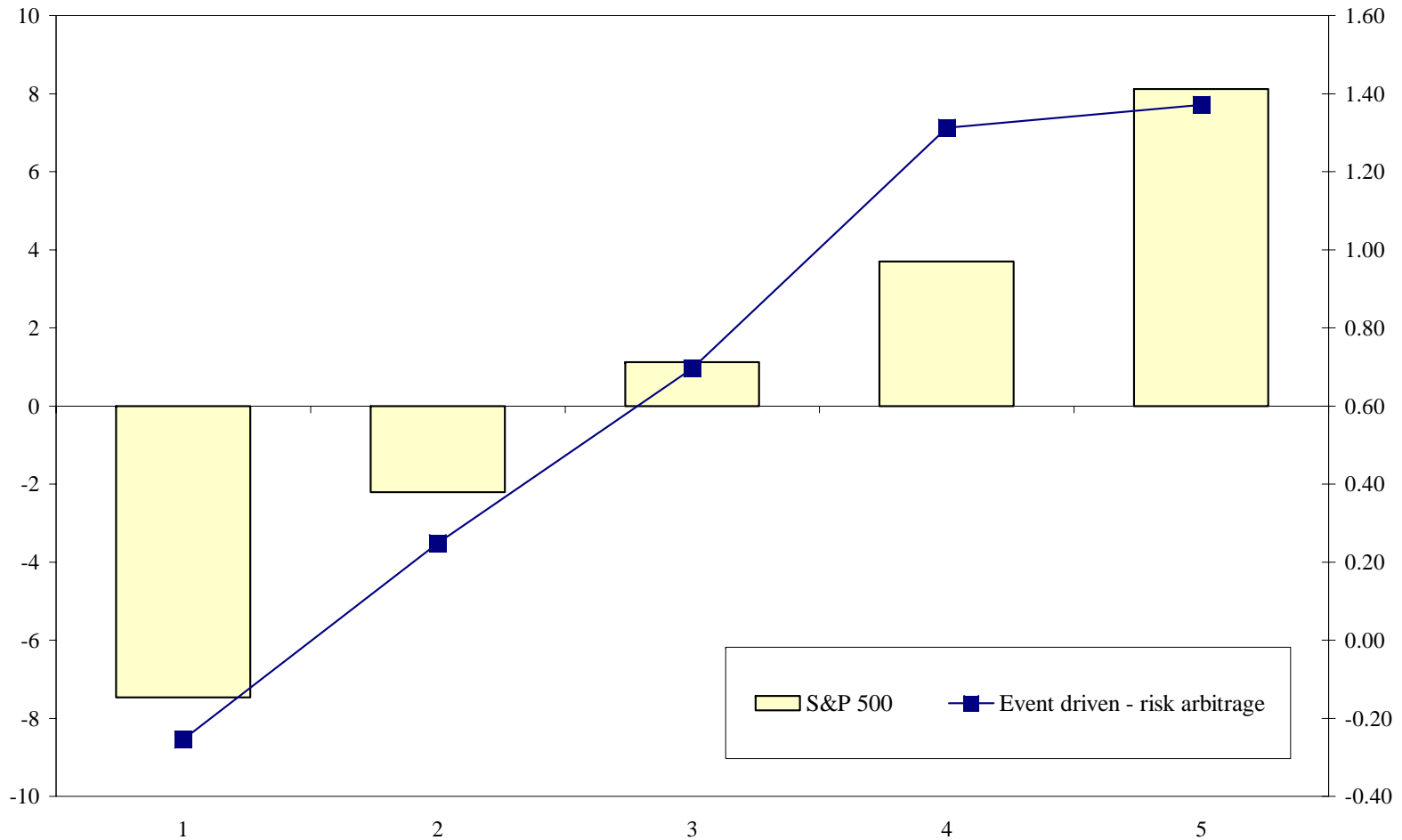
## *Non-Linear Factor Exposure – The Case of Merger Arb*

- It has been documented (Mitchel and Pulvino (JF, 2001), Aggarwal and Naik (RFS, 2004)) that merger arb performance was resembling a short position in out-of-the money put option on equity market portfolio.
- Merger arb managers go long the target and short the acquirer, which involves a risk of deal failure, which occurs more in bear market situations.



# Factor-Based Approach – Linear Case

## *Non-Linear Factor Exposure – The Case of Merger Arb*



The performance of the risk arbitrage strategy (in % p.a., right scale) versus U.S. equities (in % p.a., left scale)

# Factor-Based Approach – Linear Case

## *Non-Linear Factor Exposures*

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- Other examples of non-linear dependencies include:
  - Convertible arb returns exhibit an implicit short position in at-the-money put option on equity markets (conv arb managers lose money in case of large down moves in equity returns: delta hedge not sufficient when prices drop sharply);
  - Trend-follower CTA returns exhibit a long position in lookback straddle (which delivers the ex-post maximum payout of any trend-following strategy aiming at entering a given market at the lowest point and exiting the same market at the highest point);
  - Distressed debt and Fixed-income arb returns exhibit a short position in lookback straddle on differences in returns between High Yield bond index and Treasury bond index (convergence trading strategies, which works the opposite way compared to trend-following strategies);
  - Equity long-short and EMN returns exhibit a short position in out-of-the money put option on equity markets (L/S managers do particularly poorly in case of major down market moves).
- More generally, because of strict risk management rules, most hedge fund managers cut down their factor exposures when their risk budget (e.g., measured in terms of distance with respect to high watermark provisions) is spent.

# Factor-Based Approach – Linear Case

## *Non-Linear Factor Models*

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- Non-linear factor models
  - Heuristic attempts to introduce ad-hoc option portfolios to improve the performance of a hedge fund factor model.
    - Systematic analysis: Schneeweis and Spurgin (2000) and Agarwal and Naik (2004) (see also Fung and Hsieh (2004))
    - Specific strategies: pair trading (Gatev, Goetzmann and Rouwenhorst (2006)), event arbitrage (Mitchell and Pulvino (2001)), trend-following strategies (Fung and Hsieh (2004)) or fixed-income arbitrage (Fung and Hsieh (2002))
  - Statistical models aiming at extracting implied option payoffs from hedge fund return observations (Diez de los Rios and Garcia (2006)).

# Factor-Based Approach – Linear Case

## *A Controlled Experiment*

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- We simulate the performance of a (pseudo) hedge fund manager who rolls over a monthly short position in at-the-money puts on the S&P500 index.
- We try and replicate the strategy performance using i) a linear model and ii) a model à la Diez de los Rios and Garcia (2006).
- Out-of-sample correlations (0.884 for the linear model versus 0.997 for the option-based model) suggest a potential for improvement from the addition of the statistically estimated option position.

# Factor-Based Approach – Linear Case

## *Conditional Factor Models*

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- Conditional factor models
  - Heuristic rule-based approaches
    - “Rule-based strategies” (Jaeger and Wagner (2005))
    - “Primitive trading strategies” (Fung and Hsieh (2007))
  - Econometric approaches
    - Auto-regressive models (Bollen and Whaley (2007))
    - Regime-switching model (Billioy, Getmansky and Pelizzonx (2006)), kalman filter models, or use of instrumental variables (Basu and Stremme (2007), Kazemi and Li (2007))

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# Factor-Based Approach – Non-Linear Case

## *Introducing Non-Linear Factors*

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- The process proposed in Diez de los Rios and Garcia (2006) is based upon a statistical estimation of the non-linear function relating underlying factors to hedge fund returns, as an alternative to relying on exogenous specification of the option involved in the replication strategy.
- In particular, this approach allows one to (i) determine the portfolio of options that best approximates the returns of a given hedge fund, (ii) use options on any benchmark portfolio deemed to best characterize the strategies of the fund (and not simply traded options on an equity index), (iii) estimate the corresponding moneyness of the options that best characterize the returns of a particular fund, (iv) assess if the presence of the estimated nonlinearities are statistically significant.

# Factor-Based Approach – Non-Linear Case

## *Formal Approach*

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- We consider a regression equation that relates the excess return on a given hedge fund  $i$  to the returns of a set of factors as well as options on these factors with different strikes.

$$R_t^{HF_i} = \beta_0 + \sum_{j=1}^J \beta_{ij} F_{jt} + \sum_{j=1}^J \sum_{l=1}^{m_{ij}} \delta_{ijl} \max(F_{jt} - k_{ijl}, 0) + \varepsilon_{it}$$

- The goal is to optimally identify for each hedge fund  $i$  and each each factor  $j$  the number of options  $m_{ij}$  as well as the set of strike prices  $k_{ijl}$ , rather than setting them a priori as was done in previous studies.

# Factor-Based Approach – Non-Linear Case

## *Formal Approach – Con't*

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- While the functional form seemingly involves only call options, put options can be obtained.
- For example, a short put position is obtained as:

$$\left. \begin{array}{l} \beta_1 > 0 \\ \beta_0 = k\beta_1 \\ \delta = -\beta_1 \end{array} \right\} \Rightarrow \beta_0 + \beta_1 F_t + \delta \max(F_t - k, 0) = \underbrace{-k\beta_1 + \beta_1 F_t - \beta_1 \max(F_t - k, 0)}_{=-\beta \max(k - F_t, 0)}$$

- Similarly, linear combinations of calls and puts can also be obtained.

# Factor-Based Approach – Non-Linear Case

## *Formal Approach – Con't*

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- Considering for simplicity options on a single-factor, the return on an equity portfolio.

$$R_t^{HF} = \beta_0(k) + \beta(k)' F_{jt} + \sum_{l=1}^m \delta_l \max(R_{M,t} - k_l, 0) + \varepsilon_t$$

- Start with a single option, and run OLS regressions for all possible values of strike  $k$ .
- Avoid extreme strike values ( $k=0$  induces multi-collinearity problems with respect to linear factors, while  $k=\infty$  induces a trivial linear dependency).
- Then construct the *Wald* statistics  $T(k)$  for the null hypothesis that the model is linear ( $\delta=0$ ).

# Factor-Based Approach – Non-Linear Case

## *Formal Approach – Con't*

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- The estimated coefficients as well as the *Wald* statistics  $T(k)$  are a function of  $k$ .
- Maximizing  $T(k)$  over  $k$  values leads to the *sup Wald* statistics:

$$k^* = \arg \max_{k \in [k_{\min}, k_{\max}]} \hat{T}(k)$$

- Hansen (1996) points out that the usual chi-square distribution is not valid for  $T(k)^*$  since the a priori unknown parameter  $k$  has been determined in a data-dependent setup.
- Alternatively, Hansen (1996) proposes a simulation procedure in order to determine the asymptotic p-value of  $T(k)^*$ ; see appendix for details.

# Factor-Based Approach – Non-Linear Case

## *Empirical Results*

	Average Returns		Standard deviation		Cornish-Fisher-VaR		Sharpe-Ratio (2% p.a.)		Correlation	Info Ratio
	Index	Clone	Index	Clone	Index	Clone	Index	Clone		
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CTA Global	0.061	0.043	0.090	0.080	0.037	0.039	0.457	0.291	0.251	-0.219
Distressed Securities	0.141	-0.013	0.043	0.044	0.009	0.026	2.794	-0.753	0.109	-2.366
Emerging Markets	0.168	0.033	0.101	0.111	0.032	0.041	1.470	0.121	0.444	-1.079
Equity Market Neutral	0.082	0.025	0.019	0.018	0.001	0.006	3.358	0.294	-0.120	-1.981
Event Driven	0.117	0.003	0.046	0.040	0.014	0.021	2.124	-0.415	0.481	-2.358
Fixed Income Arbitrage	0.075	-0.034	0.018	0.062	0.002	0.036	3.107	-0.882	0.261	-1.724
Global Macro	0.091	0.025	0.051	0.047	0.013	0.023	1.372	0.095	0.460	-1.212
Long/Short Equity	0.103	0.018	0.069	0.066	0.022	0.035	1.206	-0.031	0.650	-1.407
Merger Arbitrage	0.085	0.007	0.031	0.029	0.010	0.015	2.094	-0.434	0.187	-1.902
Relative Value	0.095	0.010	0.031	0.029	0.008	0.014	2.446	-0.334	0.344	-2.299
Short Selling	-0.006	0.048	0.186	0.240	0.086	0.095	-0.140	0.115	0.635	0.251
Funds of Funds	0.095	0.012	0.052	0.048	0.011	0.025	1.430	-0.160	0.448	-1.458

*Performance and risk indicators for EDHEC Hedge Fund indices and their non-linear clones based on the option-augmented linear factor model are given. All statistics are based on out-of-sample returns observed over the period as of January 1999 through December 2006.*

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# Factor-Based Approach – Conditional Models

## *Discrete State Space Model - MRS*

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- Following Hamilton (1989) we implement a Gaussian Markov regime switching (MRS henceforth) model for the time series of Hedge Fund index returns and the dependant variables. The model may be written as:

$$R_t^{HF_i} = \beta_0(S_t) + \sum_{k=1}^K \beta_k(S_t)F_{kt} + \varepsilon_t$$

- Both factor exposures and residual volatilities depend on the state of nature; in our analysis and due to the required number of parameter estimates we introduce a two states model with the transition probability matrix P:

$$P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix}$$

# Factor-Based Approach – Conditional Models

## *Markov Regime Switching Model*

- According to the Gaussian hypothesis we write the joint likelihood function as

$$L(\theta) = \prod_{t=1}^T f(R_t | \theta) = \prod_{t=1}^T \left\{ \underbrace{\frac{\tilde{p}_{1,t}}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\varepsilon_t^2}{2\sigma^2}\right)}_{\eta_{1,t}} \Big|_{S_t=1} + \underbrace{\frac{\tilde{p}_{2,t}}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\varepsilon_t^2}{2\sigma^2}\right)}_{\eta_{2,t}} \Big|_{S_t=2} \right\}$$

- Here  $\theta$  denotes the vector of all unknown parameters, that is, factor exposures and the standard deviations for both states of nature as well as the smoothed state probabilities.
  - The smoothed state probabilities are obtained recursively from the filtered probabilities and the transition matrix (in the absence of any prior information, we initialize the probability from an equal-weighting scheme).
  - Finally, the set of parameters is obtained by maximising the logarithm of the above likelihood function.

# Factor-Based Approach – Conditional Models

## *Empirical Results*

	Average Returns		Standard deviation		Cornish-Fisher-VaR		Sharpe-Ratio (2% p.a.)		Correlation	Info Ratio
	Index	Clone	Index	Clone	Index	Clone	Index	Clone		
Convertible Arbitrage	0.096	0.010	0.037	0.024	0.012	0.011	2.032	-0.434	0.180	-1.982
CTA Global	0.061	-0.012	0.090	0.063	0.037	0.034	0.457	-0.504	0.287	-0.800
Distressed Securities	0.141	0.026	0.043	0.055	0.009	0.027	2.794	0.110	0.178	-1.619
Emerging Markets	0.168	0.007	0.101	0.106	0.032	0.052	1.470	-0.122	0.561	-1.474
Equity Market Neutral	0.082	0.026	0.019	0.012	0.001	0.004	3.358	0.539	0.181	-2.598
Event Driven	0.117	0.024	0.046	0.046	0.014	0.022	2.124	0.080	0.573	-2.020
Fixed Income Arbitrage	0.075	0.021	0.018	0.023	0.002	0.010	3.107	0.023	0.177	-1.940
Global Macro	0.091	0.011	0.051	0.040	0.013	0.019	1.372	-0.221	0.335	-1.405
Long/Short Equity	0.103	0.038	0.069	0.106	0.022	0.049	1.206	0.173	0.429	-0.615
Merger Arbitrage	0.085	0.040	0.031	0.026	0.010	0.009	2.094	0.765	0.427	-1.392
Relative Value	0.095	0.011	0.031	0.025	0.008	0.012	2.446	-0.355	0.465	-2.666
Short Selling	-0.006	-0.024	0.186	0.175	0.086	0.076	-0.140	-0.249	0.765	-0.218
Funds of Funds	0.095	0.025	0.052	0.047	0.011	0.019	1.430	0.097	0.444	-1.257

*Performance and risk indicators for EDHEC Hedge Fund indices and their conditionally linear clones based on Markov regime regressions are given. All statistics are based on out-of-sample returns observed over the period as of January 1999 through December 2006.*

# Factor-Based Approach – Economic Models

## *Discrete State Space Model - MRS*

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- The Kalman filter linearly links the observable variables with the state variables and has been shown to be the best filter among all one-sided linear filters (see Hamilton (1994)).

- The general presentation of linear the state space factor model is given by:

$$\beta_t = A\beta_{t-1} + \eta_t \text{ (Transition equation)}$$

$$R_t^{HF} = \beta_t' F_t + \varepsilon_t \text{ (Measurement equation)}$$

- The Kalman filtering technique consists of two steps, the prediction and the updating step.
  - We further suppose that  $\eta$  and  $\varepsilon$  are normally distributed with zero mean and covariance matrices H and G respectively.
  - In absence of prior information on the dynamics of the factor exposures, we suppose that A is the identity matrix.

# Factor-Based Approach – Conditional Models

## *Empirical Results*

	Average Returns		Standard deviation		Cornish-Fisher-VaR		Sharpe-Ratio (2% p.a.)		Correlation	Info Ratio
	Index	Clone	Index	Clone	Index	Clone	Index	Clone		
Convertible Arbitrage	0.096	0.031	0.037	0.025	0.012	0.007	2.032	0.445	0.214	-1.516
CTA Global	0.061	0.029	0.090	0.069	0.037	0.033	0.457	0.133	0.323	-0.383
Distressed Securities	0.141	0.062	0.043	0.039	0.009	0.010	2.794	1.085	0.339	-1.506
Emerging Markets	0.168	0.042	0.101	0.092	0.032	0.040	1.470	0.236	0.580	-1.281
Equity Market Neutral	0.082	0.038	0.019	0.010	0.001	0.002	3.358	1.740	0.165	-2.108
Event Driven	0.117	0.053	0.046	0.040	0.014	0.013	2.124	0.828	0.555	-1.443
Fixed Income Arbitrage	0.075	0.019	0.018	0.026	0.002	0.005	3.107	-0.032	0.364	-2.048
Global Macro	0.091	0.031	0.051	0.036	0.013	0.015	1.372	0.297	0.328	-1.100
Long/Short Equity	0.103	0.050	0.069	0.055	0.022	0.023	1.206	0.554	0.627	-0.918
Merger Arbitrage	0.085	0.045	0.031	0.021	0.010	0.005	2.094	1.178	0.428	-1.305
Relative Value	0.095	0.035	0.031	0.025	0.008	0.010	2.446	0.608	0.576	-2.129
Short Selling	-0.006	-0.002	0.186	0.176	0.086	0.073	-0.140	-0.123	0.749	-0.044
Funds of Funds	0.095	0.048	0.052	0.041	0.011	0.015	1.430	0.684	0.483	-0.915

*Performance and risk analysis of non-investible EDHEC Hedge Fund indices and their clones are given where Kalman Filter approach has been used to construct the clones. Various measures illustrate the quality of fit. All statistics are based on out-of-sample returns observed over the period as of January 1999 through December 2006.*

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# Factor-Based Approach – Economic Models

## *Factor Selection*

- We now turn to the economic factor selection for the specific hedge fund strategies, based on economic analysis and previous literature.

	const	S&P 500	Bond	Credit spread	Commodity	Dollar	S-L	VIX
Convertible Arbitrage	x	x*		x				x
CTA Global	x	x	x*		x			
Distressed Securities	x			x*			x	
Emerging Markets	x	x*					x	
Equity Market Neutral	x	x*					x	
Event Driven	x	x*					x	
Fixed Income Arbitrage	x	x	x	x*		x		
Global Macro	x		x*				x	
Long/Short Equity	x	x*					x	
Merger Arbitrage	x	x*					x	
Relative Value	x	x*						
Short Selling	x	x*					x	
Funds of Funds	x	x*					x	

*Factor selection according to economic criteria consistent with the academic literature. The asterix means that for the cubic and option-augmented model linear and non-linear components are included for the corresponding factor.*

# Factor-Based Approach – Economic Models

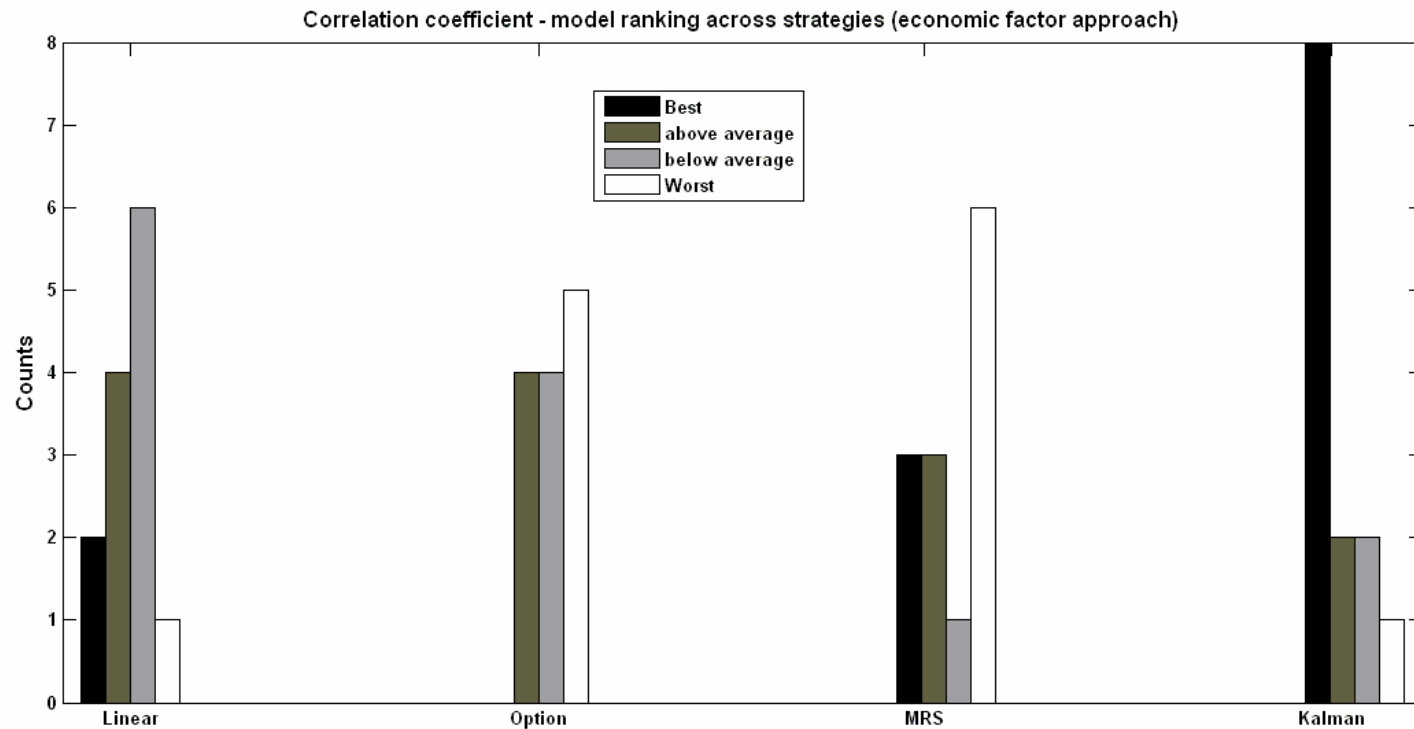
## *Empirical Results – Correlation Coefficients*

	Linear	Option	MRS	Kalman
Convertible Arbitrage	0.371	0.322	0.292	0.392
CTA Global	0.403	0.297	0.402	0.432
Distressed Securities	0.309	0.162	0.341	0.285
Emerging Markets	0.710	0.374	0.692	0.712
Equity Market Neutral	0.467	0.047	0.424	0.442
Event Driven	0.751	0.533	0.765	0.765
Fixed Income Arbitrage	0.283	0.258	0.221	0.312
Global Macro	0.388	0.075	0.396	0.363
Long/Short Equity	0.824	0.720	0.841	0.844
Merger Arbitrage	0.479	0.123	0.484	0.505
Relative Value	0.621	0.227	0.618	0.636
Short Selling	0.850	0.755	0.848	0.877
Funds of Funds	0.688	0.448	0.323	0.732

*Correlation coefficients for various replication models are compared. All statistics are based on out-of-sample returns observed over the period as of January 1999 through December 2006.*

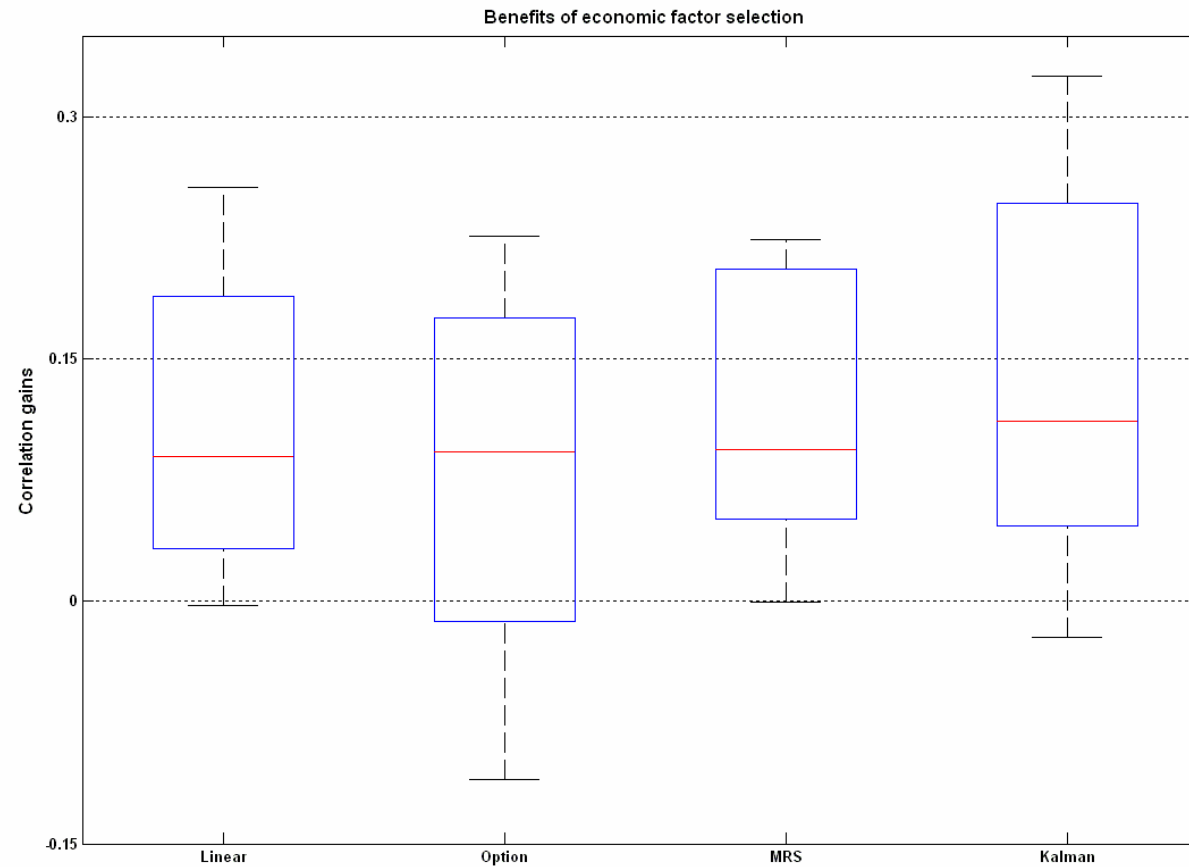
# Factor-Based Approach – Linear Case

## *Hedge Fund Indexes – Overview of the Results*



# Factor-Based Approach – Economic Models

## *Correlation Improvements*



# Factor-Based Approach – Economic Models

## *Empirical Results – Information Ratios*

	Linear	Option	MRS	Kalman
Convertible Arbitrage	-1.592	-1.620	-1.196	-1.272
CTA Global	-0.589	-0.583	-0.723	-0.439
Distressed Securities	-2.443	-2.255	-2.294	-2.355
Emerging Markets	-1.508	-1.082	-1.466	-1.472
Equity Market Neutral	-2.663	-0.775	-2.534	-2.738
Event Driven	-2.006	-2.718	-2.219	-2.120
Fixed Income Arbitrage	-2.088	-1.812	-2.523	-2.306
Global Macro	-1.301	-0.554	-1.332	-1.211
Long/Short Equity	-1.323	-0.464	-1.257	-1.407
Merger Arbitrage	-1.363	-1.830	-1.537	-1.531
Relative Value	-2.242	-2.337	-2.371	-2.428
Short Selling	-0.032	-0.625	-0.295	-0.158
Funds of Funds	-1.315	-0.948	-0.914	-1.410

*Information ratios for various replication models are compared. All statistics are based on out-of-sample returns observed over the period as of January 1999 through December 2006.*

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## Conclusion

### *Directions for Further Research*

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- A necessary condition for satisfactory hedge fund return replication is the introduction of novel econometric techniques allowing for either the statistical estimation of non-linear functions relating underlying factors to hedge fund returns, or equivalently the statistical estimation of parsimonious conditional factor models.
- While this is still very much work in progress, available results are not overly conclusive.
- Even if one is to remain cautious with respect to the prospects of hedge fund replication techniques, this area of research may generate useful insights for ex-post hedge fund performance evaluation and risk management for hedge fund portfolios.

## Conclusion

### *From HF Replication to Alternative Beta Generation*

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- Overall, we believe that so-called hedge fund replication strategies may remain as a cost-efficient way to access alternative betas through mechanical trading strategies.
- In the end, the relevant question may not be “does hedge fund replication really work?” but instead “are such solutions useful for investors?”.
- While the answer to former question is rather clearly negative, one may have more hope for a positive answer to the latter one.

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# Overview

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## Econometric Process

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- First consider a single option on the equity factor (S&P 500) and conduct OLS regressions to obtain estimates of the functional form:

$$\hat{R}_t^{HF} = \hat{\beta}_0(k) + \sum_{i=1}^I \hat{\beta}_i(k) F_{it} + \hat{\delta}(k) \max(F_{1t} - k, 0) + \varepsilon_t$$

- For valid  $k$  we compute the following Wald test-statistic:

$$\hat{T}(k) = \frac{n\delta(k)^2}{V_{\delta\delta}(k)}$$

with  $V_{\delta\delta}$  the last diagonal element of the beta-covariance matrix  $V$

$$V(k) = \left( \frac{X'X}{n} \right)^{-1} \frac{X' \varepsilon(k) \varepsilon(k)' X}{n} \left( \frac{X'X}{n} \right)^{-1}$$

# Econometric Process

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- Following Hansen (1996) and Garcia & Rios (2007), we obtain the optimal strike  $k^*$  as

$$k^* = \arg \max_{k \in [k_{\min}, k_{\max}]} \hat{T}(k)$$

where  $k_{\min}$  and  $k_{\max}$  are the 15<sup>th</sup>, respectively 85<sup>th</sup>, percentile of the empirical distribution of the underlying asset (here: S&P 500).

- We want to conclude on the significance of the option-payoff exposure  $\delta$ .
- However, since  $k^*$  is endogenously chosen, the usual distribution of the above sup Wald test statistic is not valid (chi-squared if  $k$  was known).

# Econometric Process

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- As shown in Hansen (1996) under the null-hypothesis  $\delta=0$ ,  $\hat{T}(k)$  converges in distribution towards  $T(k)$  given by:

$$T(k) = Z(k)' \left( \frac{X'X}{n} \right)^{-1} 1_\delta [V_{\delta\delta}(k)]^{-1} 1'_\delta \left( \frac{X'X}{n} \right)^{-1} Z(k)$$

where  $1_\delta$  denotes a column vector of zeros with a 1 at the last position and  $Z$  is a  $(J+2) \times 1$  gaussian process:

$$Z(k) \sim N \left( 0_{J+2}, \frac{X' \varepsilon(k) \varepsilon(k)' X}{T} \right)$$

# Econometric Process

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- Consequently, as in Hansen (1996) and Garcia & Rios (2007), perform simulations so as to compute asymptotic prob-values for the estimated non-linear factor loading  $\delta(k^*)$ .
- We simulate  $N$  Gaussian vectors  $Z_n$  and compute the corresponding values  $T_n(k)$  for all valid strikes  $k$ .
- Next, we set  $T_n^* = \max_k(T_n(k))$  and obtain the p-value for the test  $\delta=0$  as:

$$\text{prob}(\delta, k^*) = \frac{1}{N} \sum_{n=1}^N \{T_n^* \geq \hat{T}(k^*)\}$$